$5x^2 \times 6x^4$

$6x^4 = 5 \times x^2 \times 6 \times x^4$
$= 5 \times 6 \times x^2 \times x^4$
$= 30 \times x^{2+4}$
$= 30x^6$

$\times 3h^2k^4 = 4 \times h^6 \times k^5 \times 3 \times h^2 \times k^4$
$= 4 \times 3 \times h^6 \times h^2 \times k^5 \times k^4$
$= 12 \times h^{6+2} \times k^{5+4} = 12h^8k^9$

(d) $x^3y^2 \times x^6y^5 = x^3 \times y^2 \times x^6 \times y^5$
$= x^3 \times x^6 \times y^2 \times y^5$
$= x^{3+6} \times y^{2+5}$
$= x^9y^7$

Exercise 2a

the multiplication law of indices to lify each of the following.

$3^4 \times 3^2$ **(b)** $9^8 \times 9^4$
$12^5 \times 12^7$ **(d)** $x^5 \times x^3$
$c^8 \times c^6$ **(f)** $p^4 \times p^9$

the multiplication law of indices to lify each of the following.

$2x^3 \times x^7$ **(b)** $m^3 \times 5m^8$
(c) $y \times 9y^{12}$ **(d)** $3x^4 \times 5x^3$
(e) $3a \times 5a$ **(f)** $5k^4 \times 5k^4$

3. Use the multiplication law of indices to simplify each of the following.

(a) $x^3y^4 \times x^5$ **(b)** $h^7k^5 \times k^{12}$
(c) $x^3y^7 \times x^2y^4$ **(d)** $h^4k^6 \times h^5k^5$
(e) $p^6q^3 \times p^6q^3$ **(f)** $h^2k^5 \times h^5k^2$
(g) $6x^3y^5 \times x^7y^2$ **(h)** $h^4k^7 \times 8h^2k^6$

Division Law of Indices

mple 3

(a) $a^5 \div a^2$; (b) $a^7 \div a^3$.

a^2

$\dfrac{\times a \times a \times a \times a}{a \times a}$ $\dfrac{\text{5 factors}}{\text{2 factors}}$

$\times a \times a$ $5 - 2 = 3$ factors

(b) $a^7 \div a^3$

$$= \frac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a} \qquad \frac{7 \text{ factors}}{3 \text{ factors}}$$

$$= a \times a \times a \times a \qquad 7 - 3 = 4 \text{ factors}$$

$$= a^4$$

$$\text{In general, } a^m \div a^n = \frac{\overbrace{a \times a \times a \times \ldots \times a}^{m \text{ times}}}{\underbrace{a \times a \times a \times \ldots \times a}_{n \text{ times}}} \qquad \frac{m \text{ factors}}{n \text{ factors}}$$

$$= a \times a \times a \ldots \times a \qquad m - n \text{ factors}$$

$$= a^{m-n} \text{ where } m > n$$

Example 4

Simplify each of the following.

(a) $8^9 \div 8^2$

(b) $x^8 \div x^4$

(c) $12h^6 \div 4h^2$

(d) $x^3y^7 \div xy^3$

Solution

(a) $8^9 \div 8^2 = 8^{9-2}$
$= 8^7$

(b) $x^8 \div x^4 = x^{8-4}$
$= x^4$

(c) $12h^6 \div 4h^2 = \frac{{}^{3}\not{12}h^6}{\not{4}h^2} = \frac{3h^6}{h^2}$
$= 3h^{6-2}$
$= 3h^4$

(d) $x^3y^7 \div xy^3 = x^{3-1}y^{7-3}$
$= x^2y^4$

Exercise 2b

1. Simplify the following, giving your answer in index form.

(a) $8^9 \div 8^3$ **(b)** $15^{16} \div 15^8$

(c) $3^{15} \div 3^3$ **(d)** $x^7 \div x^4$

(e) $n^6 \div n$ **(f)** $y^{12} \div y^6$

2. Simplify each of the following.

(a) $8x^4 \div 2$ **(b)** $16x^7 \div 4x^3$

(c) $28y^9 \div 4y^3$ **(d)** $21y^{14} \div 3y^7$

(e) $32x^5 \div 4x^3$ **(f)** $36x^{12} \div 6x^6$

3. Simplify each of the following.

(a) $\frac{x^2y^5}{xy^2}$ **(b)** $\frac{6x^5y^7}{3x^2y^4}$

(c) $\frac{20p^{10}q^8}{5p^5q^4}$ **(d)** $49h^7k^4 \div 7hk^2$

(e) $36x^8y^6 \div 9x^6y$ **(f)** $18x^{18}y^{16} \div 2x^6y^3$

Exercise 2c

1. Express each of the following in its simplest index form.
   (a) $(5^3)^4$ (b) $(6^4)^5$
   (c) $(n^9)^4$ (d) $(y^8)^7$

2. Express each of the following in its simplest form.
   (a) $(a^3)^4 \times a^7$ (b) $y^3 \times (y^4)^2$
   (c) $x \times (x^4)^5$ (d) $(x^5)^4 \div x^3$
   (e) $(t^2)^3 \div t$ (f) $(h^5)^5 \div h^{12}$
   (g) $(a^3)^6 \div (a^2)^4$ (h) $(x^7)^5 \div (x^4)^8$

3. Simplify each of the following giving your answer in its simplest index form.
   (a) $(3x)^3$ (b) $(x^2y)^3$
   (c) $(5x^3)^2$ (d) $(x^5y^4)^4$
   (e) $(3hk^2)^3$ (f) $(7a^5b^7)^2$

4. Express each of the following as a power of a single number.
   (a) $2^5 \div 3^5$ (b) $7^4 \div 3^4$
   (c) $14^5 \div 7^5$ (d) $3^9 \times 5^9$
   (e) $4^8 \times 7^8$ (f) $5^7 \times 6^7$

5. Simplify each of the following, giving your answer in index form.
   (a) $(8x^4)^2 \times 2x^5$
   (b) $81x^{10} \div (3x^2)^3$
   (c) $2(a-b)^9 \times (a-b)^6$
   (d) $(32ab^3)^2 \div 64ab^5$
   (e) $15(2a+b)^{12} \div 3(2a+b)^3$
   (f) $(a^2b^3)^5 \times (3ab^2)^3$
   (g) $(4x^5y^4)^3 \div (2x^3y^2)^5$
   (h) $\dfrac{(2x^2y)^3}{(10xy^3)^2} \times \dfrac{(5xy^4)^3}{4xy}$
   (i) $\dfrac{8x^8y^4}{(2xy^2)^2} \times \dfrac{(4x^2y^2)^2}{(3xy)^2}$
   (j) $\dfrac{(2xy^2)^5}{(4x^2y)^2(xy^3)}$

Zero and Negative Indices

The laws for positive integral indices can be extended so that we can give meanings to zero and negative integral indices.

Example 10

Simplify (a) $5^3 \div 5^3$; (b) $8^2 \div 8^2$

(c) $\left(\frac{2}{3}\right)^{-2} \times \left(\frac{5}{4}\right)^{0} = \frac{1}{\left(\frac{2}{3}\right)^{2}} \times 1$

$= \frac{1}{\frac{4}{9}}$

$= \frac{9}{4}$

$= 2\frac{1}{4}$

(d) $(3^{-2})^{3} \times (9^{-3})^{-2} = 3^{-6} \times 9^{6}$

$= 3^{-6} \times (3^{2})^{6}$

$= 3^{-6} \times 3^{12}$

$= 3^{-6+12}$

$= 3^{6}$

$= 729$

Explorations

In (c) above, we can also get: $\left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^{2}} = \left(\frac{1}{\frac{2}{3}}\right)^{2}$

$= \left(\frac{3}{2}\right)^{2}$

From this, can you deduce any general formula for the negative powers of a fraction?

Try more questions to check your conclusion.

Exercise 2d

1. Simplify the following, giving your answers in positive indices only:

(a) $3^{7} \times 3^{-12}$ **(b)** $8^{9} \div 8^{15}$

(c) $9^{8} \div 9^{-4}$ **(d)** $6^{0} \div 6^{5}$

(e) $7^{-4} \times 7^{-5}$ **(f)** $(5^{-4})^{3}$

(g) $(9^{-2})^{-4}$ **(h)** $(a^{2})^{-3}$

(i) $(a^{-2})^{4} \div a^{3}$ **(j)** $2a^{-5} \div 7b^{-5}$

2. Simplify the following, giving your answers in negative indices only:

(a) $6^4 \times 6^{-2}$
(b) $7^8 \div 7^4$
(c) $5^0 \times 5^{-4}$
(d) $(3^{-2})^{-4}$
(e) $(8^{-4})^6$
(f) $a^2 \times b^3$
(g) $ab^2 \div ab^3$
(h) $(a^2b)^3$
(i) $a^2 \div a^5$
(j) $abc \div a^5b^4c^2$

3. Evaluate each of the following:

(a) 4^{-2}
(b) 3^{-4}
(c) $\left(\frac{-1}{4}\right)^2$
(d) $\left(\frac{2}{3}\right)^{-3}$
(e) $\left(\frac{2}{9}\right)^0$
(f) $\left(\frac{-2}{5}\right)^{-3}$
(g) $3^2 \times 4^{-3}$
(h) $(3^2)^5 \div 9^3 \times 27^{-1}$
(i) $\left(\frac{3}{4}\right)^{-1} \times \left(\frac{3}{7}\right)^0$
(j) $\left(\frac{2}{7}\right)^{-3} \times 49^{-1}$

4. Simplify the following:

(a) $a^3 \times a^0$
(b) $a^2 \times a^{-5}$
(c) $x^7 \div x^{-5}$
(d) $x^{-2} \div x^{-5}$
(e) $x^{-3} \div x^2$
(f) $x^{-4} \div x^{-7} \div x^2$
(g) $a^{500} \div a^{-600}$
(h) $(x^0)^{-7}$
(i) $(x^2yz)^4 \div (xyz)^7$
(j) $(pqr^2)^{-2} \div (p^2r^2q)^{-5}$

Fractional Indices

Earlier, we have seen that the laws of indices hold true for integral indices. We shall now find a meaning for a^n, where n is a fraction or rational number, and a is positive. We certainly hope that all the laws of indices also hold true for rational indices.

First we consider $a^{\frac{1}{2}}$.

Law I holds true for fractional indices. Hence we have

$$a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a^1 = a$$

By definition, $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = (a^{\frac{1}{2}})^2$.

Therefore, $(a^{\frac{1}{2}})^2 = a$.

Exercise 2g

1. Express the following in the form 10^n, where n is an integer.

(a) $\dfrac{10^{12}}{10^{-6} \div 10^{-6}}$ (b) $\dfrac{10^{9} \times 10^{-7}}{10^{-5}}$

(c) $\dfrac{10^{-4}}{10^{-7} \times 10^{-3}}$ (d) $\dfrac{10^{-6} \times 10^{-7}}{10^{-14} \times 10^{2}}$

(e) $\dfrac{10^{-3} \times 10^{15}}{10^{-7} \div 10^{-28}}$ (f) $\dfrac{10^{12} \div 10^{-9}}{10^{-7} \div 10^{-16}}$

(i) $\dfrac{3.6 \times 10^{4}}{10^{3}}$ (j) $\dfrac{6.55 \times 10^{-2}}{10^{-3}}$

2. Express the following numbers in the standard form.

(a) 912 400 (b) 28 000 000
(c) 0.043 5 (d) 0.000 77
(e) 0.008 306 (f) 0.296
(g) 74.8 (h) 70 600

3. Express the following in ordinary notation.

(a) 6.37×10^3 (b) 4.213×10^{-3}
(c) 8.1×10^{-5} (d) 1.729×10^4
(e) 3.82×10^{-1} (f) 9.8×10^6
(g) $3(4.7 \times 10^{-2})$ (h) $0.7(1.2 \times 10^3)$

4. Evaluate and then express $2(11 \times 10^3)^2$ in the form $A \times 10^n$ where $1 \leq A < 10$ and n is an integer.

5. Evaluate and then express $21(3.0 \times 10^2) \div (7.0 \times 10^3)$ in the standard form.

6. Evaluate each of the following, giving your answer in standard form and the units in brackets.

(a) 78 microseconds + 512 nanoseconds (seconds)
(b) 583 picoseconds + 2.5 nanoseconds (seconds)
(c) 1.35 microseconds – 47 nanoseconds (seconds)
(d) 4.57 centimetres – 87 micrometres (metres)
(e) 0.75 millimetres – 4.7 micrometres (metres)
(f) 25 nanometres – 89 picometres (metres)

Use of Calculator

The examples below show how indices and numbers in the standard form are expressed and evaluated using a scientific calculator.

1. To find the value of a number raised to a power, we use the $\boxed{x^y}$ key.

For example,

(a) to find 2^7, press 2 $\boxed{x^y}$ 7 $\boxed{=}$ to get the answer 128.

(b) to find 5.4^3, press 5.4 $\boxed{x^y}$ 3 $\boxed{=}$ to get the answer 157.464.

(c) to find 4^{-2}, press 4 $\boxed{x^y}$ $\boxed{+/-}$ 2 $\boxed{=}$ to get the answer 0.062 5.