Factoring Trinomials

Multiplying Polynomials

What is the result when you multiply (x+3)(x+2)?

Using algebra tiles, we have

x + 3

x

\+

2

The resulting trinomial is $x^2 + 5x + 6$.

Using the distributive property, we have:

$$\begin{aligned}(x+3)(x+2) &= x(x+2) + 3(x+2)\\ &= x^2 + 2x + 3x + 6\\ &= x^2 + 5x + 6\end{aligned}$$

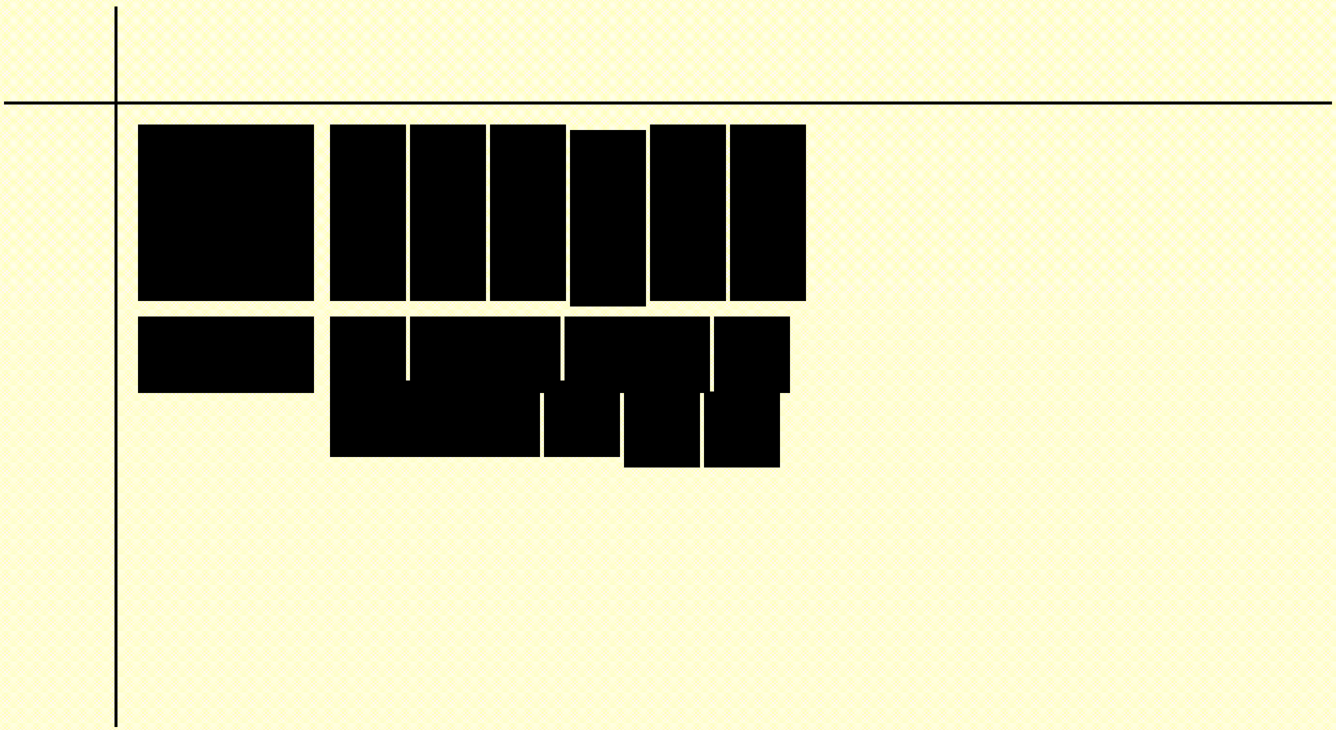
The resulting trinomial is $x^2 + 5x + 6$.

Notice that 2 + 3 = 5 which is the coefficient of the middle term.
2 x 3 = 6 which is the value of the constant. The coefficient of x^2 is 1 and 1 x 6 = 6 which is again the value of the constant.

Factoring Trinomials

How can we factor trinomials such as $x^2 + 7x + 12$?

One method would be to again use algebra tiles.



Start with the x^2.
Add the twelve tiles with a value of 1.

Try to complete the rectangle using the 7 tiles labeled x.

Factoring Trinomials

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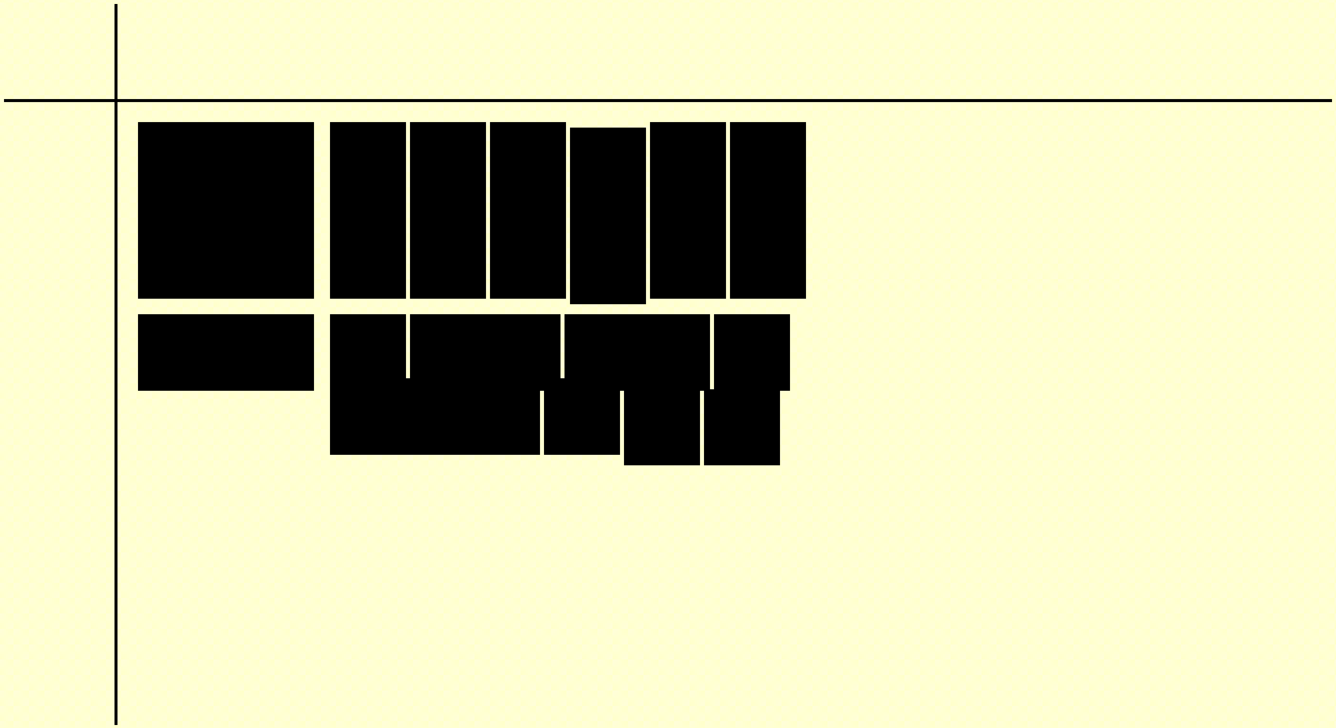

Note that we have used 7 tiles with "x", but are still short one "x". Thus, we must rearrange the tiles with a value of 1.

Factoring Trinomials

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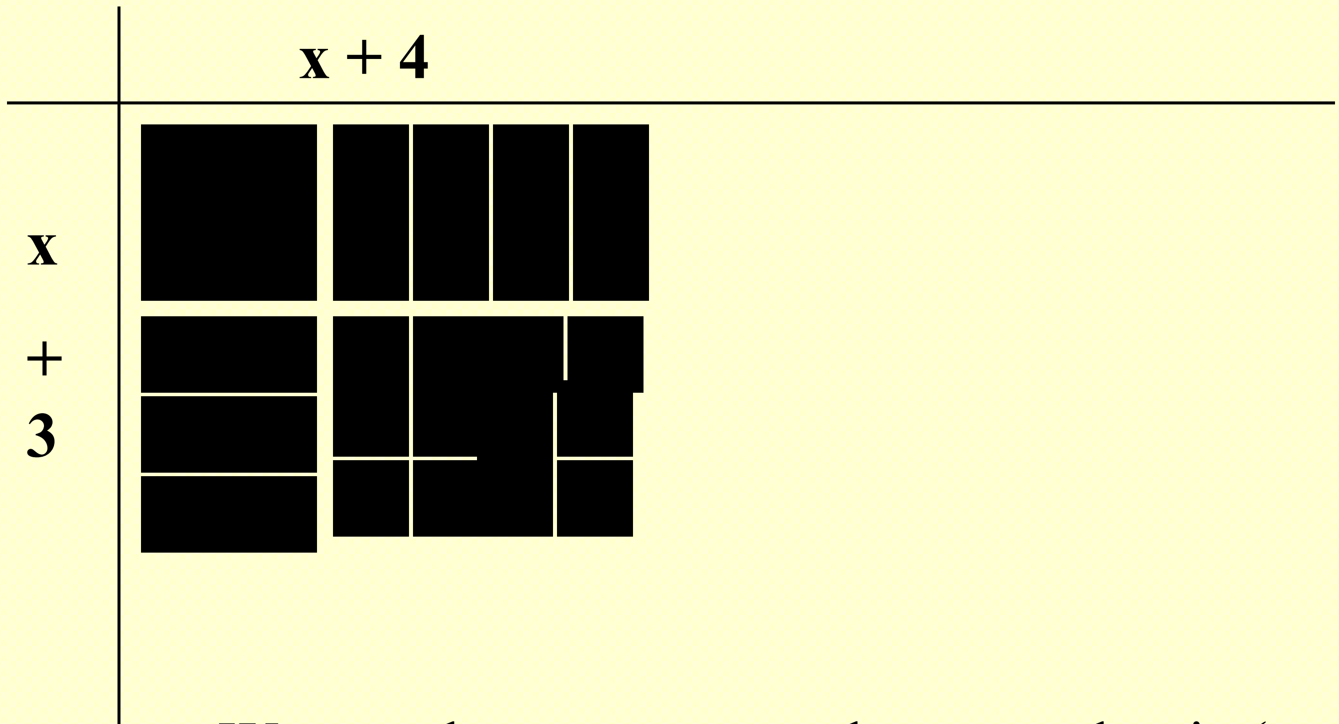

We now have a rectangular array that is (x+4) by (x+3) units.

Therefore, $x^2 + 7x + 12 = \boxed{(x + 4)(x + 3)}$.

Factoring Trinomials

While the use of algebra tiles helps us to visualize these concepts, there are some drawbacks to this method, especially when it comes to working with larger numbers and the time it takes for trial and error.

Thus, we need to have a method that is fast and efficient and works for factoring trinomials.

In our previous example, we said that $x^2 + 7x + 12 = (x + 4)(x + 3)$.

Step 1: Find the factors of the coefficient of the term with x^2.

1 x 1

Step 2: Find the factors of the constant.

1 x 12   2 x 6   3 x 4

Step 3: The trinomial $ax^2 + bx + c = (mx + p)(nx + q)$ where

$a = m \cdot n$, $c = p \cdot q$,   and $b = mq + np$

Factoring Trinomials

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$$a = m \cdot n, \quad c = p \cdot q, \quad \text{and} \quad b = mq + np$$

Step 4: Write trial factors and check the middle term.

$(x + 1)(x + 12)$	$x + 12x = 13\,x$	No
$(x + 2)(x + 6)$	$2x + 6x = 8x$	No

Factoring Trinomials

Step 4: Write trial factors and check the middle term.

$(x + 1)(x + 12)$	$x + 12x = 13x$	No
$(x + 2)(x + 6)$	$2x + 6x = 8x$	No
$(x + 3)(x + 4)$	$3x + 4x = 7x$	Yes

This method works for trinomials which can be factored. However, it also involves trial and error and may be somewhat time consuming.

Alternative Method

Trinomials are written as $ax^2 + bx + c$. However, a, b, and c may be positive or negative. Thus a trinomial may actually appear as:

$$ax^2 + bx + c$$

$$ax^2 - bx + c$$

$$ax^2 - bx - c$$

$$ax^2 + bx - c$$

Alternative Method

<u>Case 1</u>: If a = 1, b is positive, and c is positive, find two numbers whose product is c and whose sum is b.

<u>Example</u> $x^2 + 10x + 16$

a = 1, b = 10, c = 16

The factors of 16 are 1 and 16, 2 and 8, 4 and 4.

2 + 8 is 10.

$$\boldsymbol{x^2 + 10x + 16 = x^2 + 2x + 10x + 16}$$

Write 10x as the sum of the two factors.

$$\boldsymbol{= (x^2 + 2x) + (8x + 16)}$$

Use parentheses to group terms with common factors.

$$\boldsymbol{= x(x + 2) + 8(x + 2)}$$

$$\boldsymbol{= (x + 8)(x + 2)}$$

Factor

Apply the distributive property.

Alternative Method

Case 2: If a = 1, b is positive and c is negative, find two numbers whose product is c and whose difference is b.

Example $x^2 + 5x - 14$

$a = 1$, $b = 5$, $c = -14$

The factors of –14 are –1 and 14, 1 and –14, -2 and 7, and 2 and –7. $-2 + 7 = 5$.

$$\boldsymbol{x^2 + 5x - 14 = x^2 - 2x + 7x - 14}$$

Write 5x as the sum of the two factors.

$$\boldsymbol{= (x^2 - 2x) + (7x - 14)}$$

Use parentheses to group terms with common factors.

$$\boldsymbol{= x(x - 2) + 7(x - 2)}$$

Factor

$$\boldsymbol{= (x + 7)(x - 2)}$$

Apply the distributive property.

Alternative Method

<u>Case 3</u>: If a = 1, b is negative and c is positive, find two numbers whose product is c and whose sum is b.

<u>Example</u> $x^2 - 13x + 36$

$a = 1, \; b = -13, \; c = 36$

The factors of 36 are 1 and 36, 2 and 18, 3 and 12, 4 and 9,

-1 and –36, -2 and –28, -3 and –12, -4 and –9. $-4 + (-9) = -13$

$$x^2 - 13x + 36 = x^2 - 4x - 9x + 36$$

Write -13x as the sum of the two factors.

$$= (x^2 - 4x) + (-9x + 36)$$

$$= x(x - 4) + (-9)(x - 4)$$

Use parentheses to group terms with common factors.

$$= (x - 9)(x - 4)$$

Factor

Apply the distributive property.

Alternative Method

Case 4: If a = 1, b is negative and c is negative, find two numbers whose product is c and whose sum is b.

Example $x^2 - 8x - 20$

$a = 1,\ b = -8,\ c = -20$

The factors of -20 are 1 and -20, -1 and 20, 2 and -10, -2 and 10, 4 and –5, and –4 and 5. 2 + (-10) = -8.

$$x^2 - 8x - 20 = x^2 + 2x - 10x - 20$$

Write -8x as the sum of the two factors.

$$= (x^2 + 2x) - (10x + 20)$$

Use parentheses to group terms with common factors.

$$= x(x + 2) - 10(x + 2)$$

Factor

$$= (x - 10)(x + 2)$$

Apply the distributive property.

Alternative Method

<u>Case 5</u>: If a 🕒 1, find the 🕖ac🕖. If c is positive, find two factors of 🕖ac🕖whose sum is b.

<u>Example</u> $6x^2 + 13x + 5$

a = 6, b = 13, c = 5, 🕖ac🕖=30

The factors of 30 are 1 and 30, 2 and 15, 3 and 10, 5 and 6,

-1 and –30, -2 and –15, -3 and –10, -5 and –6. 3 + 10 = 13.

$$6x^2 + 13x + 5 = 6x^2 + 3x + 10x + 5$$

Write 13x as the sum of the two factors.

$$= (6x^2 + 3x) + (10x + 5)$$

Use parentheses to group terms with common factors.

$$= 3x(2x + 5) + 5(2x + 1)$$

Factor

$$= (3x + 5)(2x + 1)$$

Apply the distributive property.

Alternative Method

<u>Case 6</u>: If a 🕒 1, find the 🕖ac🕖. If c is negative, find two factors of 🕖ac🕖whose difference is b.

<u>Example</u> $8x^2 + 2x - 15$

a = 8, b = 2, c = -15, 🕖ac🕖= 120

The factors of -120 are 1,120,2,60,3,40,4,30,5,26,6,20,8,15,10,12.

12 – 10 = 2.

$$\mathbf{8x^2 + 2x - 15 = 8x^2 + 12x - 10x - 15}$$

Write 2x as the sum of the two factors.

$$\mathbf{= (8x^2 + 12x) + (-10x - 15)}$$

Use parentheses to group terms with common factors.

$$\mathbf{= 4x(2x + 3) - 5(2x + 3)}$$

Factor

$$\mathbf{= (4x - 5)(2x + 3)}$$

Apply the distributive property.

You Try It

Factor each trinomial if possible.

1) $t^2 - 4t - 21$

2) $x^2 + 12x + 32$

3) $x^2 - 10x + 24$

4) $x^2 + 3x - 18$

5) $2x^2 - x - 21$

6) $3x^2 + 11x + 10$

7) $x^2 - 2x + 35$

Problem 1

$t^2 - 4t - 21$

a = 1, b = -4, c = -21

The factors of –21 are –1,21, 1,-21, -3,7, 3,-7.

3 + (-7) = -4.

$$\begin{aligned} t^2 - 4t - 21 &= t^2 + 3t - 7t - 21 \\ &= (t^2 + 3t) + (-7t - 21) \\ &= t(t+3) - 7(t+3) \\ &= (t-7)(t+3) \end{aligned}$$

Problem 2

$x^2 + 12x + 32$

a = 1, b = 12, c = 32

The factors of 32 are 1,32, 2,16, 4,8.

4 + 8 = 32

$$
\begin{aligned}
x^2 + 12x + 32 &= x^2 + 4x + 8x + 32 \\
&= (x^2 + 4x) + (8x + 32) \\
&= x(x + 4) + 8(x + 4) \\
&= (x + 8)(x + 4)
\end{aligned}
$$

Problem 3

$x^2 - 10x + 24$

a = 1, b = -10, c = 24

The factors of 24 are 1,24, 2,12, 3,8, 4,6, -1,-24, -2,-12, -3,-8, -4,-6

-4 + (-6) = -10

$$\begin{aligned} x^2 - 10x + 24 &= x^2 - 4x - 6x + 24 \\ &= (x^2 - 4x) + (-6x + 24) \\ &= x(x - 4) - 6(x - 4) \\ &= (x - 6)(x - 4) \end{aligned}$$

Problem 4

$x^2 + 3x - 18$

a = 1, b = 3, c = -18

The factors of -18 are 1,-18, -1,18, 2,-9, -2,9, 3,-6, -3,6

-3 + 6 = 3

$$\begin{aligned} x^2 + 3x - 18 &= x^2 - 3x + 6x - 18 \\ &= (x^2 - 3x) + (6x - 18) \\ &= x(x-3) + 6(x-3) \\ &= (x+6)(x-3) \end{aligned}$$

Problem 5

$2x^2 - x - 21$

a = 2, b = -1, c = -21, 🕖ac🕖=42

The factors of 42 are 1,42, 2,21, 3,14, 6,7.

6 – 7 = -1

$$\begin{aligned} 2x^2 - x - 21 &= 2x^2 + 6x - 7x - 21 \\ &= (2x^2 + 6x) + (-7x - 21) \\ &= 2x(x+3) - 7(x+3) \\ &= (2x-7)(x+3) \end{aligned}$$

Problem 6

$3x^2 + 8x + 5$

a = 3, b = 8, c = 5, 🕖ac🕖=15

The factors of 15 are 1,15, 3,5.

3 + 5 = 8

$$\begin{aligned}
3x^2 + 8x + 5 &= 3x^2 + 3x + 5x + 5 \\
&= (3x^2 + 3x) + (5x + 5) \\
&= 3x(x+1) + 5(x+1) \\
&= (3x+5)(x+1)
\end{aligned}$$

Problem 7

$x^2 - 2x + 35$

$a = 1$, $b = -2$, $c = 35$,

The factors of 35 are 1,35, -1,-35, 5,7, and –5,-7

None of these pairs of factors gives a sum of –2.

Therefore, this trinomial can’t be factored by this method.